Hypergraph *p*-Laplacian Regularization for Remotely Sensed Image Recognition

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*Abstract***— Graph-based and manifold-regularization (MR) based semisupervised learning, including Laplacian regularization (LapR) and hypergraph LapR (HLapR), have achieved prominent performance in preserving locality and similarity information. However, it is still a great challenge to exactly explore and exploit the local structure of the data distribution. In this paper, we present an efficient and effective approximation algorithm of hypergraph** *p***-Laplacian and then propose hypergraph** *p***-LapR (HpLapR) to preserve the geometry of the probability distribution. In particular, hypergraph is a generalization of a standard graph while hypergraph** *p***-Laplacian is a nonlinear generalization of the standard graph Laplacian. The proposed HpLapR shows great potential to exploit the local structures. We integrate HpLapR with logistic regression for remote sensing image recognition. Experiments on UC-Merced data set demonstrate that the proposed HpLapR has superior performance compared with several popular MR methods including LapR and HLapR.**

*Index Terms***— Hypergraph, manifold learning, remote sensing, semisupervised learning (SSL),** *p***-Laplacian.**

NOMENCLATURE

- *V* Finite set of vertices.
- w(*e*) Weight associated with each hyperedge *e*.
- δ(*e*) Degree of a hyperedge *e* ∈ *E*, $δ(e) = |e|$.
 L^{hp} Hypergraph Laplacian.
- *L*^{hp} Hypergraph Laplacian.
 \mathcal{F}^{*hp} Eigenvectors of hypergraph
- Eigenvectors of hypergraph *p*-Laplacian $\mathcal{F}^{*hp} = (f^{*hp1}, f^{*hp2}, \ldots, f^{*hpn}).$

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- $|| f ||_1^2$ *^I* Appropriate penalty term corresponding to the probability distribution.
- **f** = $[f(x_1), f(x_2), \ldots, f(x_{l+u})]^T$.

$$
\Upsilon_I
$$
 Parameter corresponds with $||f||_I^2$.

pre(*r*) Precision at recall *r*.

I. INTRODUCTION

NOWADAYS, billions of images (e.g., action images, human face images, object images, and scene images) are uploaded to social media platform such as YouTube, Facebook, and Twitter. Image recognition including scene recognition [1]–[3], human face recognition [4], [5], action recognition [6]–[8], and remotely sensed image recognition [9]–[12] has become a quite significant topic in machine vision. With the fast development of space technologies, remotely sensed techniques have been utilized in many applications including targeting, environment monitoring, surveillance, and military systems. In the remotely sensed community, it is particularly important to effectively assign the categories of high-resolution remotely sensed imagery data, e.g., aerial images and radar images. Aerial scene classification [13], 14] has received growing attention due to the drastically increasing number of aerial images and the highly complex geometrical structures and spatial patterns.

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However, because annotating images is costly and time consuming, a small number of labeled samples are available in practical applications, whereas a lot of unlabeled samples are easy to collect. Semisupervised learning (SSL) which can make use of labeled and unlabeled data has been investigated to solve this problem. One successful work is manifold regularization (MR), which has attracted considerable attention due to its rich theoretical studies [15]–[17] and its excellent performance in multimedia data (e.g., text, image, video, and audio) processing [18]–[24]. The main idea of MR is to explore the geometry of the intrinsic probability distribution of data to leverage the learning performance. Another successful work is graph-based SSL [25], [26], which constructs a similarity graph over data to exploit the local geometry of labeled and unlabeled data and has achieved appealing perfor-

in practice. The MR framework [15] exploits the geometry of the probability distribution of the data and incorporates it as a regularization term. Laplacian regularization (LapR) is one prominent MR-based SSL algorithm, which determines the underlying manifold by using the graph Laplacian. Wang *et al.* [27] presented a manifold-regularized (MR) multiview subspace clustering method to better incorporate the correlated and complementary information from different views. The graph Laplacian is constructed to maintain the local data manifold of each view. Luo *et al.* [19] employed MR to smooth the functions along the data manifold for multitask learning. Jiang *et al.* [18] presented a mutimanifold method for recognition by exploring the local geometric structure of samples. Liu and Tao [28] proposed multiview Hessian-regularized logistic regression which combines multiple Hessian regularizations to leverage the local geometry. Lu and Mou [29] built a model of sparse-feature-selection-based MR to select the optimal information and preserve the underlying manifold structure of data for scene recognition.

mance due to its flexibility and low computation complexity

Typically, in graph-based SSL, it is assumed that there is a graph over the data lying on data manifolds. In the graph, vertices represent samples and edge weights indicate the similarity between samples. For example, Zhou *et al.* [30] constructed a directed graph learned from labeled and unlabeled data for web categorization, in which each vertex represents a web page, and each edge represents a hyperlink between two web pages. For graph-based SSL, it is essential to construct an effective graph over data with complex distribution. Compared with the existing simple graph that only models the pairwise relationship of images, hypergraph learning that uses a hyperedge to link multiple samples can model the high-order relationship of samples.

In [31], the hypergraph idea was first introduced to the field of computer vision. It is a generalization of a simple graph. Unlike a simple graph that considers the relationship between two vertices, a set of vertices is connected by a hyperedge in a hypergraph. Thus, the hypergraph contains more local grouping information than does the simple graph. Hypergraph has been widely used in image classification [32], [4], ranking [33], [34], and video segmentation [35]. Sun *et al.* [36] constructed a hypergraph to exploit the correlation information among different labels for multilabel learning. Zass and Shashua [37] presented a hypergraph-based image matching problem in a probabilistic setting represented by a convex optimization problem. Huang *et al.* [34] proposed a hypergraph-based transductive algorithm to the field of image retrieval. Yu *et al.* [4] proposed an adaptive hypergraph learning method for transductive image classification.

In this paper, we propose a hypergraph *p*-Laplacianregularized (HpLapR) method for remote sensing image recognition. The hypergraph and *p*-Laplacian [38], [39], [41] both provide convincing theoretical evidences to better preserve the local structure of data. However, the computation of hypergraph *p*-Laplacian is a strenuous task. We provide an effective and efficient approximation algorithm of hypergraph *p*-Laplacian. Considering the high-order relationship of samples, the HpLapR is built for preserving local structures. HpLapR is also introduced to logistic regression for remote sensing image recognition. Experiments on the UC-Merced data set [42] compare the proposed method with the popular algorithms including LapR, hypergraph LapR (HLapR), and *p*-LapR (pLapR). The contributions of this paper can be summarized as follows.

- 1) We present an efficient approximation algorithm of hypergraph *p*-Laplacian, significantly improving computation efficiency.
- 2) We propose HpLapR to preserve the local similarity of data.
- 3) We integrate HpLapR into logistic regression and conduct comprehensive experiments to empirically analyze our method on UC-Merced data set. The experimental results validate the effectiveness of our method.

The rest of this paper is organized as follows. Section II briefly reviews related work on MR and hypergraph learning. Section III introduces an approximate computation of the hypergraph *p*-Laplacian. Section IV proposes HpLapR. Section V presents the HpLapR logistic regression. Section VI provides the experimental results and analysis on UC-Merced data set. Finally, Section VII gives the conclusion.

II. RELATED WORK

In this section, we briefly review MR and hypergraph.

A. Manifold Regularization

Assume that the estimated function is generated from the probability distribution on samples (labeled and unlabeled samples). The labeled samples are (x, y) pairs generated according to probability distribution, and lie on the estimation curve in the ideal case. The unlabeled samples are drawn according to the marginal distribution. Based on the manifold assumption that if two samples are close in the intrinsic geometry, they have the similar labels. It is important to exploit the knowledge of the marginal distribution for better function learning.

Introducing an additional regularizer for preserving local structures, the MR framework can be interpreted as regularization algorithms that consist of different empirical cost functions, complexity measures in an appropriately chosen reproducing kernel Hilbert space (RKHS) and additional information about the geometric structure of the margins. Consider *l* labeled samples and *u* unlabeled samples, the objective function can be written as

$$
f^* = \arg\min_{f \in H_K} \frac{1}{l} \sum_{i=1}^l V(x_i, y_i, f) + \Upsilon_A \|f\|_K^2 + \Upsilon_I \|f\|_I^2 \tag{1}
$$

where H_K is the RKHS, V is a loss function, such as the hinge loss function max[0, 1 – $y_i f(x_i)$] for support vector machines. There is an associated RKHS H_K of functions $X \to \mathbb{R}$ with the corresponding norm $\|\|_k$. $\|f\|_K^2$ is used to control the complexity of the classification model, while $||f||_I^2$ is an appropriate penalty term corresponding to the probability distribution. Parameters ϒ*A* and ϒ*I* control the complexity of the function in the ambient space and the intrinsic geometry, respectively.

The classical Representer theorem states that f exists in H_K and can be written as

$$
f^*(x) = \sum_{i=1}^{l+u} \alpha_i^* K(x_i, x)
$$
 (2)

where α^{i} is the coefficient and *K* is a kernel function.

Graph Laplacian has been widely used to explore and exploit the local geometry of data distribution. As a nonlinear generalization of the standard graph Laplacian, graph *p*-Laplacian has attracted attentions from the machine-learning community. Zhou and Schölkopf [39] proposed a general discrete regularization framework of *p*-Laplacian for the classification problem, and its objective function can be computed as follows:

$$
f^* = \underset{f \in \mathcal{H}(V)}{\text{argmin}} \ \{ \mathcal{S}_p(f) + \mu \| f - y \|^2 \} \tag{3}
$$

where $\mathcal{S}_p(f) := (1/2) \sum_{v \in V} ||\nabla_v f||^p$ is the *p*-Dirichlet form of the function f , μ is a parameter balancing the two competing terms, and $y \in \{-1, 0, 1\}$ depends on the label of the sample.

Bühler and Hein [38] used the graph *p*-Laplacian for spectral clustering and demonstrated the relationship between the second eigenvalue of the graph *p*-Laplacian and the optimal Cheeger cut as follows:

$$
\text{RCC} \le \text{RCC}^* \le p \left(\max_{i \in V} d_i \right)^{\frac{p-1}{p}} \text{RCC}^{\frac{1}{p}} \tag{4}
$$

or

$$
\text{NCC} \leq \text{NCC}^* \leq p\text{NCC}^{\frac{1}{p}} \tag{5}
$$

where RCC[∗] and NCC[∗] are the ratio/normalized Cheeger cut values obtained by thresholding the second eigenvector of the unnormalized/normalized *p*-Laplacian, *di* is the degree of vertex *i*, and RCC and NCC are the optimal ratio/normalized Cheeger cut values.

Luo *et al.* [43] used the *p*-Laplacian for multiclass clustering and provided an approximation of the whole eigenvectors by solving the tractable optimization problem

$$
\min_{\mathcal{F}} J_E(\mathcal{F}) = \sum_{k} \frac{\sum_{ij} w_{ij} |f_i^k - f_j^k|^p}{\|f^k\|_p^p}
$$
\ns.t.

\n
$$
\mathcal{F}^T \mathcal{F} = I
$$
\n(6)

where w_{ij} is the edge weight, f^k is an eigenvector of *p*-Laplacian, and $\mathcal{F} = (f^1, f^2, \dots, f^n)$ are whole eigenvectors.

Liu *et al.* [6] proposed pLapR sparse coding for preserving the manifold structure.

B. Hypergraph

In machine-learning issues, we generally assume pairwise relationship among the object set. An object set endowed with pairwise relationship can be considered as a graph. The graph can be undirected or directed. However, in a number of questions, it is not complete to represent the relations among samples only using simple graphs. Hypergraph learning [34] addresses this problem. Compared with traditional graph, a hypergraph illustrates the complex relationship by hyperedges which connect three or more vertices (see in Fig. 1).

Let *V* denote a finite set of vertices and *E* be a family of subsets of *V* such that $\bigcup_{e \in E} = V$. A hypergraph $G = (V, E)$ corresponding to the vertex set *V* and the hyperedge set *E*. Denote the weight associated with each hyperedge *e* as $w(e)$. The degree of a vertex $v \in V$ is defined by $d(v) = \sum_{\{e \in E \mid v \in e\}} w(e)$. The degree of a hyperedge $e \in E$ is denoted by $\delta(e) = |e|$. Denote the incident matrix *H* by a $|V| \times |E|$ matrix, whose entry $h(v, e) = 1$ if $v \in e$, and $h(v, e) = 0$ otherwise. Then

$$
d(v) = \sum_{e \in E} w(e)h(v, e)
$$
 (7)

$$
\delta(e) = \sum_{v \in V} h(v, e). \tag{8}
$$

Let *Dv* denote the diagonal matrices containing the degree of vertex, *De* denote the diagonal degree matrices of each hyperedge, and *W* is the diagonal matrix of edge weights. Then, the hypergraph Laplacian can be defined.

There have been many methods for building the graph Laplacian of hypergraphs across the literature. The first category includes star expansion [44], clique expansion [44], and Rodriquez's [45] Laplacian. These methods aim to construct a simple graph from the original hypergraph, and then partition the vertices using spectral clustering techniques. The second category of approaches defines a hypergraph Laplacian using analogies from the simple graph Laplacian. Representative methods in this category include Bolla's [46] Laplacian and Zhou *et al.*'s [47] normalized Laplacian. In [47], the normalized hypergraph Laplacian is defined as

$$
L^{\rm hp} = I - Dv^{-1/2} \text{HW} D_e^{-1} H^T D v^{-1/2}.
$$
 (9)

Note that L^{hp} is the positive semidefinite. The adjacency matrix of hypergraph can be formulated as follows:

$$
W^{\rm hp} = \text{HW}H^T - Dv. \tag{10}
$$

Fig. 1. Block scheme of hypergraph. (Left) Simple graph in which two points are joined together by an edge if they are highly similarity. A hypergraph completely illustrates the complex relationship among points by hyperedges. (Right) *H* matrix of the hypergraph. The entry (v_i, e_j) is set to 1 if a hyperedge e_j contains v*i* , or 0 otherwise.

For a simple graph, the edge degree matrix *De* is replaced by 2*I*. Thus, the standard graph Laplacian is

$$
L = I - \frac{1}{2}Dv^{-\frac{1}{2}}HWH^T Dv^{-\frac{1}{2}}
$$

= $\frac{1}{2}(I - Dv^{-1/2}W^{hp} Dv^{-1/2}).$ (11)

III. APPROXIMATION OF HYPERGRAPH *p*-LAPLACIAN

In this section, we introduce the approximation algorithm of hypergraph p -Laplacian L_p^{hp} .

Assume that hypergraph *p*-Laplacian has *n* eigenvectors $\mathcal{F}^{*hp} = (f^{*hp1}, f^{*hp2}, \dots, f^{*hpn})$ associated with unique eigenvalues $\lambda^{*hp} = (\lambda_1^{*hp}, \lambda_2^{*hp}, \dots, \lambda_n^{*hp})$, we compute the approximation of L_p^{hp} by $L_p^{\text{hp}} = \mathcal{F}^{* \text{hp}} \lambda^{* \text{hp}} \mathcal{F}^{* \text{hp}}$. Thus, it is important to obtain all eigenvectors and eigenvalues of hypergraph *p*-Laplacian.

Although a complete analysis of hypergraph *p*-Laplacian is challenging, we can easily generate a hypergraph with a group of hyperedges [47]. In detail, we construct hypergraph Laplacian L^{hp} and compute adjacency matrix W^{hp} by (8) and (9), respectively.

Then, we introduce the basic definition of *p*-Laplacian Δ_p^w including its eigenvalue and eigenvector.

The real number λ_p is called an eigenvalue of Δ_p^w , if there exists a function $f: V \to \mathcal{R}$ satisfying the relationship as follows:

$$
\left(\Delta_p^w f\right)_i = \lambda_p \phi_p(f_i), \quad i \in V. \tag{12}
$$

Function *f* is called a *p*-eigenfunction (also called eigenvector) associated with λ_p . ϕ_p is defined by $\phi_p(x)$ = $|x|^{p-1}$ sign(*x*). Note that the operator $\Delta_2^w = L$ becomes the regular graph Laplacian.

Following previous studies on *p*-Laplacian [38], eigenvalue and the corresponding eigenvector on nonlinear operator Δ_p^w can be computed by the theorem.

Function F_p has a critical point at f if and only if f is an eigenvector of Δ_p^w . F_p is defined as

$$
F_p(f) = \frac{\sum_{ij} w_{ij} |f_i - f_j|^p}{2 \|f\|_p^p}
$$
 (13)

where

$$
||f||_p^p = \sum_i |f_i|^p.
$$

Here, w_{ij} is the edge weight and the corresponding eigenvalue λ_p is given by $\lambda_p = F_p(f)$. The above theorem serves as the foundational analysis of eigenvectors and eigenvalues. Moreover, $F_p(\alpha f) = F_p(f)$ applies for any real value of α .

Naturally, we can extend the above theorem to the hypergraph *p*-Laplacian as follows.

f hp is an eigenvector of hypergraph *p*-Laplacian, if and only if the following function F_p^{hp} has a critical point at f^{hp} :

$$
F_p^{\text{hp}}(f^{\text{hp}}) = \frac{\sum_{ij} w_{ij}^{\text{hp}} |f_i^{\text{hp}} - f_j^{\text{hp}}|^p}{2 \|f^{\text{hp}}\|_p^p}
$$
(14)

where

$$
\|f^{\text{hp}}\|_p^p = \sum_i |f_i^{\text{hp}}|^p.
$$

The eigenvalue λ^{hp} associated with f^{hp} is given by $\lambda^{\text{hp}} = F_p^{\text{hp}}(f^{\text{hp}})$.

If we want to obtain all eigenvectors and eigenvalues of hypergraph *p*-Laplacian, we have to find all critical points of function F_p^{hp} . Following this idea, we can obtain the full eigenvector space by solving local solution of the following optimization problem:

$$
\min_{\mathcal{F}^{\text{hp}}} J(\mathcal{F}^{\text{hp}}) = \sum_{k} F_{p}^{\text{hp}}(f_{i}^{\text{hp}k})
$$
\n
$$
\text{s.t. } \sum_{i} \phi_{p}(f_{i}^{\text{hp}k}) \phi_{p}(f_{i}^{\text{hp}l}) = 0, \quad k \neq l \tag{15}
$$

where $\mathcal{F}^{\text{hp}} = (f^{\text{hp1}}, f^{\text{hp2}}, \dots, f^{\text{hpn}}).$

We analyze the full eigenvectors by solving the following hypergraph *p*-Laplacian embedding problem instead of (15):

$$
\min_{\mathcal{F}^{\text{hp}}} J_E(\mathcal{F}^{\text{hp}}) = \sum_k \frac{\sum_{ij} w_{ij}^{\text{hp}} |f_i^{\text{hp}k} - f_j^{\text{hp}k}|^p}{\|f^{\text{hp}k}\|_p^p}
$$
\n
$$
\text{s.t. } \mathcal{F}^{\text{hp}T} \mathcal{F}^{\text{hp}} = I. \tag{16}
$$

Differentiating with respect to f_i^{hpk} yields the following equation:

$$
\frac{\partial J_E}{\partial f_i^{\text{hpk}}} = \frac{1}{\|f^{\text{hpk}}\|_p^p} \left[\sum_j w_{ij}^{\text{hp}} \phi_p \left(f_i^{\text{hpk}} - f_j^{\text{hpk}} \right) - \frac{\phi_p \left(f_i^{\text{hpk}} \right)}{\|f^{\text{hpk}}\|_p^p} \right].
$$
\n(17)

Solving problem (16) with the gradient descend optimization, the gradient is defined in the following way:

$$
G^{\rm hp} = \frac{\partial J_E}{\partial \mathcal{F}^{\rm hp}} - \mathcal{F}^{\rm hp} \left(\frac{\partial J_E}{\partial \mathcal{F}^{\rm hp}}\right)^T \mathcal{F}^{\rm hp}.\tag{18}
$$

Meanwhile, the full eigenvalue $\lambda^{\text{hp}} = (\lambda_1^{\text{hp}}, \lambda_2^{\text{hp}}, \ldots, \lambda_n^{\text{hp}})$ can be computed by

$$
\lambda_k^{\text{hp}} = \frac{\sum_{ij} w_{ij}^{\text{hp}} |f_i^{\text{hp}k} - f_j^{\text{hp}k}|^p}{\|f^{\text{hp}k}\|_p^p}.
$$
 (19)

Finally, the approximation of L_p^{hp} can be solved by the full eigenvectors and eigenvalues of hypergraph *p*-Laplacian in this paper. We summarize the approximation of hypergraph *p*-Laplacian in Algorithm 1. In the algorithm, the step length α is set to $\alpha = 0.01((\sum_{ik} |\mathcal{F}_{ik}^{\text{hp}}|)/(\sum_{ik} |G_{ik}^{\text{hp}}|)).$

IV. HPLAPR

In SSL, we are given *N* training samples including *l* labeled samples $\{(x_i, y_i)\}_{i=1}^{\overline{l}}$ and *u* unlabeled samples $\{(x_j)\}_{j=l+1}^{l+u}$. Class labels are given in $Y = \{y_i\}_{i=1}^l$, where $y_i \in \{\pm 1\}$. Typically, $l \ll u$ and the goal is to predict the labels of unseen samples.

[∂]F*h p* is

Step2: Decompose graph Laplacian: $L^{hp} = USU^T$. **Initialize**: $\mathcal{F}^{hp} = U(:, 1:K)$

Step3: While not converged do:
\n
$$
G^{hp} = \frac{\partial J_E}{\partial \mathcal{F}^{hp}} - \mathcal{F}^{hp} \left(\frac{\partial J_E}{\partial \mathcal{F}^{hp}} \right)^T \mathcal{F}^{hp}
$$
, where $\frac{\partial J_E}{\partial \mathcal{F}^{hp}}$
\ngiven by Equation (16)
\n
$$
\mathcal{F}^{hp} = \mathcal{F}^{hp} - \alpha G^{hp}
$$
\nEnd
\nStep4: $\lambda_k^{hp} = \frac{\sum_{ij} w_{ij}^{hp} |f_i^{hpk} - f_j^{hpk}|^p}{\|f_i^{hpk}\|_p^p}$
\nreturn: $L_p^{hp} = \mathcal{F}^{hp} \lambda^{hp} \mathcal{F}^{hp}$

According to the MR framework, the proposed HpLapR can be written as the following optimization problem:

$$
f^* = \arg \min_{f \in H_K} \frac{1}{l} \sum_{i=1}^l V(x_i, y_i, f) + \Upsilon_A \|f\|_K^2 + \frac{\Upsilon_I}{(l+u)^2} \mathbf{f}^T L_p^{\text{lp}} \mathbf{f}. \tag{20}
$$

Here, **f** is given as $\mathbf{f} = [f(x_1), f(x_2), \dots, f(x_{l+u})]^T$, and L_p^{hp} is the hypergraph *p*-Laplacian.

Next, to solve the optimization problem (20), we discuss the Representer theorem of HpLapR.

Lemma 1: Suppose an arbitrary finite set of points ${x_1, \ldots, x_n}$, the kernel function $K(x, y)$ satisfying $K(x, y) =$ $K(y, x)$ is a positive semidefinite kernel, and kernel matrix K with $K_{ij} = K(x_i, x_j)$ is symmetric positive definite. If the penalty term of the optimization problem (1) is a strictly monotonically increasing real-valued function on f , the minimizer of the problem admits an expansion in terms of both labeled and unlabeled samples as

$$
f^*(x) = \sum_{i=1}^{l+u} \alpha_i^* K(x_i, x).
$$
 (21)

Lemma 2: The hypergraph *p*-Laplacian L_p^{hp} is positive semidefinite [48].

Theorem 3: The minimization of the object problem (20) with respect to *f* has the representation

$$
f^*(x) = \sum_{i=1}^{l+u} \alpha_i^* K(x_i, x).
$$
 (22)

Proof: Kernel *K* and hypergraph *p*-Laplacian L_p^{hp} are both positive semidefinite. Thus, the regularization term $\Upsilon_A ||f||_K^2 +$ $(\Upsilon_I/(l+u)^2)$ **f**^T L_p^{hp} **f** is a monotonically increasing real-value function with respect to f . According to Lemma 1, the proof of Theorem 3 is complete.

The Representer theorem demonstrates that the solution of (20) exists and has the general form of (22).

V. HPLAPR LOGISTIC REGRESSION

The proposed HpLapR can be applied to variant applications by integrating different choices of loss function $V(x_i, y_i, f)$ into MR framework. In this section, we apply HpLapR to logistic regression and give its complexity analysis.

Substitute logistic loss function into (20), the HpLapR can be rewritten as

$$
f^* = \arg\min_{f \in H_K} \frac{1}{l} \sum_{i=1}^l (\log(1 + e^{-y_i f(x_i)})) + \Upsilon_A \|f\|_K^2 + \frac{\Upsilon_I}{(l+u)^2} \mathbf{f}^T L_p^{\text{hp}} \mathbf{f}^T. \tag{23}
$$

According to the Representer theorem, the solution of (23) with respect to f exists and can be expressed by (22). Thus, we finally construct the HpLapR as the following optimization

Fig. 2. Framework of HpLapR for remote sensing image classification.

Fig. 3. Class examples of UC-Merced data set. The data set totally has 21 remote sensing categories that can be simply grouped into six groups according to the distinction of land use. Each column represents one group.

problem:

$$
f^* = \arg\min_{f \in H_K} \frac{1}{l} \sum_{i=1}^l (\log(1 + e^{-y_i K(x_i, x)a}))
$$

$$
+ \Upsilon_A \alpha^T K \alpha + \frac{\Upsilon_I}{(l+u)^2} \alpha^T K L_p^{\text{hp}} K \alpha. \quad (24)
$$

To solve the optimization problem in (24), we can employ the conjugate gradient algorithm. We take derivative of the objective function as

$$
\nabla f(\alpha)
$$
\n
$$
= -\frac{\log(e)}{l} \sum_{i=1}^{l} \left(\frac{y_i}{1 + e^{y_i K(x_i, x)\alpha}} \mathbf{K}^T(x_i, x) \right)
$$
\n
$$
+ \Upsilon_A (\mathbf{K} + \mathbf{K}^T) \alpha + \frac{\Upsilon_l}{(l+u)^2} (\mathbf{K} L_p^{\text{hp}} \mathbf{K} + (\mathbf{K} L_p^{\text{hp}} \mathbf{K})^T) \alpha. \quad (25)
$$

The optimization procedure of conjugate gradient algorithm for HpLapR logistic regression is described in Algorithm 2.

Suppose that we are given *N* samples. Denote the embedding dimension as *K* and the number of iterations as η_1 for approximation of hypergraph *p*-Laplacian. The time cost for constructing hypergraph *p*-Laplacian is $O(\eta_1(N^2K + nK^2))$. When K is much smaller than N , the time cost is around $O(\eta_1 N^2)$. Denote the number of iterations as η_2 for HpLapR logistic regression and the number of candidate

Algorithm 2 HpLapR Logistic Regression

Input: *l* labeled samples $\{(x_i, y_i)\}_{i=1}^l$, *u* unlabeled samples $\{(x_j)\}_{j=l+1}^{l+u}$. **output:** Estimated function: $f^*(x) = \sum_{i=1}^n a_i^* K(x_i, x)$. Step1: Construct approximate Hypergraph p-Lapalcian L_p^{hp} . Step2: Choose a kernel function and compute the Gram matrix $\mathbf{K}_{ij} = \mathbf{K}(x_i, x_j)$. Step3: Compute α^* : **Initialize:** $\alpha^0 \in \mathbb{R}^N$, $d^0 = -\nabla f(\alpha)$, $\delta, 0 < \varepsilon \ll 1$, $m = 0$ **while** $|f(a^{m+1}) - f(a^m)| > \varepsilon$ **do:** $\alpha^{m+1} = \alpha^m + \delta d^m$ $d^{m+1} = -\nabla f\left(\alpha^{m+1}\right) + \frac{\|\nabla f(a^{m+1})\|^2}{\|\nabla f(a^m)\|^2}d^m$ $m = m + 1$ **return:** $\alpha^* = \alpha^{m+1}$

parameters that need the m-fold cross-validation as *r*. The time cost for HpLapR logistic regression is $O(\eta_2 r N^3)$.

VI. EXPERIMENTS

In this section, to evaluate the effectiveness of the proposed HpLapR, we compare HpLapR with other local structure preserving algorithms including LapR, HLapR, and pLapR. We apply the logistic regression for remote sensing image

Fig. 4. Performance of mAP with different *p* values on validation set.

Fig. 5. mAP performance of different algorithms.

classification. Fig. 2 illustrates the framework of HpLapR for UC-Merced data set.

UC-Merced data set [42] consists of totally 2100 landuse images collected from aerial orthoimage with the pixel resolution of one foot. The original images were downloaded from the United States Geological Survey National Map of 20 U.S. regions. These images were manually selected into 21 classes: agricultural, airplane, baseball diamond, beach, buildings, chaparral, dense residential, forest, freeway, golf course, harbor, intersection, medium density residential,

Fig. 6. AP performance of different methods on several classes.

mobile home park, overpass, parking lot, river, runway, sparse residential, storage tanks, and tennis courts. In this paper, we organize these 21 classes into six groups (see Fig. 3). Note that UC-Merced data set contains a variety of land-use classes, which make the data set quite challenging, especially several highly overlapped classes, e.g., sparse residential, medium density residential, and dense residential. They mainly differ in the density.

In our experiments, we extract high-level visual features using the deep convolutional neural network [49]. We randomly choose 50 images per class as training samples and the rest as testing samples. For hypergraph construction, we regard each sample in the training set as a vertex, and generate a hyperedge for each vertex with its *k* nearest neighbors (so the hyperedge connects $k + 1$ samples) [34]. It is worth noting that for our experiments, the *k*NN-based hyperedges generating method is implemented only in six groups, not in the overall training samples. For example, for a sample of baseball diamond, the vertices of the corresponding hyperedge are chosen from the first group (baseball diamond, golf course, and tennis courts) of Fig. 3.

In semisupervised classification experiments, we assign 10%, 20%, 30%, and 50% samples of training data as labeled data; the rest are used as unlabeled data. The process is repeated five times independently to avoid any bias introduced by the random partition of data.

We conduct the experiments on the data set to obtain the proper modal parameters. The neighborhood size *k* of a hypergraph varies in a range $\{5, 6, 7, \ldots, 15\}$ through

Fig. 7. PR curve of different methods with 10% labeled samples.

cross-validation. Regularization parameters γ*A* and γ *I* are selected from the candidate set $\{10^i \mid i\}$ $-10, -9, -8, \ldots, 10$ } through cross-validation, and parameter *p* for pLapR and HpLapR is chosen from $\{1, 1.1, 1.2, \ldots, 3\}$ through cross-validation with 10% labeled samples on the training data, respectively. We verify the classification performance by average precision (AP) performance for single class and mean AP (mAP) [50] for overall classes. The AP is defined as the mean precision at a set of 11 equally

Fig. 8. Confusion matrices of different methods with 10% labeled samples. (a) LapR. (b) HLapR. (c) pLapR. (d) HpLapR.

spaced recall levels and can be expressed as follows:

$$
AP = \frac{1}{11} \sum_{t} \left[\max_{r \ge t} \text{pre}(r) \right], \quad t \in \{0, 0.1, 0.2, \dots, 1.0\} \quad (26)
$$

where $pre(r)$ is the precision at recall r. The mAP is the mean AP over all remote sensing image classes and can be written as

$$
mAP = \frac{\sum_{i=1}^{c} AP_i}{c}
$$
 (27)

where *c* is the number of remote sensing image classes.

Fig. 4 illustrates the mAP performance of pLapR and HpLapR on the validation set when *p* varies. The *x*-axis is parameter *p* and the *y*-axis is mAP for performance measure. We can see that the best mAP performance for pLapR can be obtained when $p = 2.3$, while the best performance of HpLapR is achieved when *p* is equal to 2.6.

We compare our proposed HpLapR with the representative LapR, HLapR, and pLapR. From Fig. 5, we can observe that HpLapR outperforms other methods especially when only a small number of samples are labeled. This suggests that our proposed method has the superiority to preserve the local structure of the data because it integrates hypergraph learning with *p*-Laplacian.

To evaluate the effectiveness of HpLapR for single class, Fig. 6 shows the AP results of different methods on several selected land-use classes including beach, dense residential, freeway, and tennis court. From Fig. 6, we can find that in most cases, HpLapR performs better than do both pLapR and HLapR, while pLapR and HLapR consistently outperforms than LapR.

Moreover, we show the precision–recall (PR) curves and the confusion matrices of different methods with the 10% labeled samples in Figs. 7 and 8, respectively. We can find that our proposed HpLapR can obtain the better performance for the most classes in comparison to other methods. We compute the *f* -measure values of LapR, HLapR, pLapR, and HpLapR. The HpLapR can get the higher *f* -measure value (0.695859) while those of LapR, HLapR, and pLapR are 0.674872, 0.677027, and 0.679709, respectively.

VII. CONCLUSION

Existing SSL algorithms have achieved great performance in computer vision applications including classification, clustering, and ranking. However, it is still challenging on how to obtain the high-order relationship while exploiting the local geometry of the data distribution. Therefore, after introducing a full approximation algorithm of hypergraph *p*-Laplacian to significantly low down its computation complexity, this paper has proposed an HpLapR method to preserve the geometry of the probability distribution. This is because hypergraph and *p*-Laplacian have the advantage of preserving local structures. Furthermore, we proposed HpLapR logistic regression for remote sensing recognition. The experimental results on UC-Merced data set have demonstrated the effectiveness of our proposed method in comparison to other regularized methods including LapR, HLapR, and pLapR.

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